International Journal of Scientific & Engineering Research, Volume 4, Issue 11, November-2013 1185
ISSN 2229-5518

Particular and unique solutions of DGLAP evolution equation in leading order and gluon structure function at small-x

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Abstract: We present particular and unique solutions of Dokshitzer- Gribov- Lipatov- Altarelli-Parisi (DGLAP) evolution equation for gluon structure function in leading order (LO) and obtain t and x-evolutions of gluon structure function at small-x. The results are compared with a recent global parameterization.

Keywords: Altarelli-Parisi equation, Taylor expansion, complete solution particular solution, structure function, small-x physics, unique solution.
PACS Nos.: 12.38.*Bx*, 12.39.-*x*, 13.60.*Hb*

1. Introduction

In recent papers [1-2], particular and unique solutions of the Dokshitzer- Gribov- Lipatov- Altarelli- Parisi (DGLAP) [3-6] evolution equations for t and x-evolutions of singlet and non-singlet structure functions in leading order (LO) and next-to-leading order (NLO) at small-x have been reported. The same technique can be applied to the DGLAP evolution equations for gluon structure function in LO to obtain t and x-evolutions of gluon structure function. These LO results are compared with a recent global parameterization [7-8]. Here Section 1, Section 2, and Section 3 will give the introduction, the necessary theory and the results and discussion respectively.

2. Theory

The DGLAP evolution equation for gluon structure function has the standard form [9] as

$$\frac{\partial G(x,t)}{\partial t}-\frac{A_f}{t}\left\{\left(\frac{11}{12}-\frac{N_f}{18}+\ln(1-x)\right)G(x,t)+I_g\right\}=0, \tag{1}$$

where

$$I_g=\int_x^1 dw\left[\begin{array}{l}\dfrac{wG(x/w,t)-G(x,t)}{1-w}+\left(w(1-w)+\dfrac{1-w}{w}\right)G(x/w,t)\\ +\dfrac{2}{9}\left(\dfrac{1+(1-w)^2}{w}\right)F_2^S(x/w,t)\end{array}\right], \tag{2}$$

$t=\ln\left(Q^2/\Lambda^2\right)$, $\quad A_f=\dfrac{36}{33-N_f}$, $\quad N_f$ being the number of flavour.

Let us introduce the variable $u = 1-w$ and note that [10]

$$\frac{x}{w}=\frac{x}{1-u}=x\sum_{k=0}^{\infty}u^k. \tag{3}$$

The series (3) is convergent for $|u|<1$. Since $x<w<1$, so $0<u<1-x$ and hence the convergence criterion is satisfied. Now, using Taylor expansion method [11] we can rewrite $G(x/w, t)$ as

$$G(x/w,t)=G\left(x+x\sum_{k=1}^{\infty}u^k,t\right)$$

$$=G(x,t)+x\sum_{k=1}^{\infty}u^k\frac{\partial G(x,t)}{\partial x}+\frac{1}{2}x^2\left(\sum_{k=1}^{\infty}u^k\right)^2\frac{\partial G(x,t)}{\partial x^2}+\ldots.. \quad 4)$$

which covers the whole range of u, $0<u<1-x$. Since x is small in our region of discussion, the terms containing x^2 and higher powers of x can be neglected as our first approximation as discussed in our earlier works [1-2, 12-14] and $G(x/w, t)$ can be approximated for small-x as

$$G(x/w,t)\cong G(x,t)+x\sum_{k=1}^{\infty}u^k\frac{\partial G(x,t)}{\partial x}. \tag{5}$$

Similarly, $F_2^S(x/w, t)$ can be approximated for small-x as

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$$F_2^S(x/w,t) \cong F_2^S(x,t) + x\sum_{k=1}^{\infty} u^k \frac{\partial F_2^S(x,t)}{\partial x}. \qquad (6)$$

Using equations (5) and (6) in equations (1) and (2) and performing u-integrations we get

$$\frac{\partial G(x,t)}{\partial t} - \frac{A_f}{t}\left[\begin{array}{l} A_1(x)F_2^S(x,t) + B_1(x)\dfrac{\partial F_2^S(x,t)}{\partial x} \\ +C_1(x)G(x,t) + D_1(x)\dfrac{\partial G(x,t)}{\partial x}\end{array}\right] = 0, \qquad (7)$$

where

$$A_1(x) = -\left[\frac{2}{9}(1-x) + \frac{1}{9}(1-x)^2 + \frac{4}{9}\ln x\right],$$

$$B_1(x) = x\left[\frac{4}{9x} + \frac{4}{9}(1-x) + \frac{1}{9}(1-x)^2 + \frac{8}{9}\ln x - \frac{4}{9}\right],$$

$$C_1(x) = \left(\frac{11}{12} - \frac{N_f}{18}\right) + \ln(1-x) - \left[\begin{array}{l} 2(1-x) - \frac{1}{2}(1-x)^2 \\ +\frac{1}{3}(1-x)^3 + \ln x\end{array}\right],$$

$$D_1(x) = x\left[\frac{1}{x} + 2(1-x) + \frac{1}{3}(1-x)^3 + 2\ln x - 1\right].$$

For simplicity we assume [1-2]

$G(x, t) = K(x)\, F_2^S(x, t)$, where $K(x)$ is a function of x. Therefore

$$F_2^S(x,t) = K_1(x)G(x,t), \text{ where } K_1(x) = 1/K(x). \qquad (8)$$

Now equation (7) becomes

$$\frac{\partial G(x,t)}{\partial t} - \frac{A_f}{t}\left[P(x)G(x,t) + Q(x)\frac{\partial G(x,t)}{\partial x}\right] = 0, \qquad (9)$$

where $P(x) = A_1(x)K_1(x) + B_1(x)\dfrac{\partial K_1(x)}{\partial x} + C_1(x)$ and $Q(x) = B_1(x)K_1(x) + D_1(x)$.

The general solutions of equations (9) is [11, 15] $F(U, V) = 0$, where F is an arbitrary function and $U(x, t, G) = C_1$ and $V(x, t, G) = C_2$ form a solution of equations

$$\frac{dx}{A_f Q(x)} = \frac{dt}{-t} = \frac{dG(x,t)}{-A_f P(x)G(x,t)}. \qquad (10)$$

Solving equation (10) we obtain

$$U(x,t,G) = t\exp\left[\frac{1}{A_f}\int\frac{1}{Q(x)}dx\right]$$

and $V(x,t,G) = G(x,t)\exp\left[\int\dfrac{P(x)}{Q(x)}dx\right]$.

If U and V are two independent solutions of equation (10) and if α and β are arbitrary constants, then $V = \alpha U + \beta$ may be taken as a complete solution of equation (10). Now the complete solution [13-14]

$$G(x,t)\exp\left[\int\frac{P(x)}{Q(x)}dx\right] = \alpha t\exp\left[\frac{1}{A_f}\int\frac{1}{Q(x)}dx\right] + \beta \qquad (11)$$

is a two-parameter family of surfaces, which does not have an envelope, since the arbitrary constants enter linearly [11]. Differentiating equation (11) with respect to β we get 0 = 1, which is absurd. Hence there is no singular solution. The one parameter family determined by taking $\beta = \alpha^2$ has equation

$$G(x,t)\exp\left[\int\frac{P(x)}{Q(x)}dx\right] = \alpha t\exp\left[\frac{1}{A_f}\int\frac{1}{Q(x)}dx\right] + \alpha^2. \qquad (12)$$

Differentiating equation (12) with respect to α, we get $\alpha = -\dfrac{1}{2}t\exp\left[\dfrac{1}{A_f}\int\dfrac{1}{Q(x)}dx\right]$. Putting the value of α in equation (12), we obtain the envelope

$$G(x,t) = -\frac{1}{4}t^2\exp\left[\int\left(\frac{2}{A_f Q(x)} - \frac{P(x)}{Q(x)}\right)dx\right], \qquad (13)$$

which is merely a particular solution of the general solution. Now, defining

$G(x,t_0) = -\dfrac{1}{4}t_0^{\,2}\exp\left[\int\left(\dfrac{2}{A_f Q(x)} - \dfrac{P(x)}{Q(x)}\right)dx\right]$, at $t = t_0$, where $t_0 = \ln(Q_0^2/\Lambda^2)$ at any lower value $Q = Q_0$, we get from equation (13)

$$G(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^2, \qquad (14)$$

which gives the t-evolution of gluon structure function $G(x, t)$. Again defining,

$G(x_0,t) = -\dfrac{1}{4}t^2\exp\left[\int\left(\dfrac{2}{A_f Q(x)} - \dfrac{P(x)}{Q(x)}\right)dx\right]_{x=x_0}$, we obtain from equation (13)

$$G(x,t) = G(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{2}{A_f Q(x)} - \frac{P(x)}{Q(x)}\right)dx\right] \qquad (15)$$

which gives the x-evolution of gluon structure function $G(x, t)$.

For the complete solution of equation (9), we take $\beta = \alpha^2$ in equation (11). If we take $\beta = \alpha$ in equation (11) and differentiating with respect to α as before, we get $0 = t\exp\left[\frac{1}{A_f}\int\frac{1}{Q(x)}dx\right]+1$ from which we can not determine the value of α. But if we take $\beta = \alpha^3$ in equation (11) and differentiating with respect to α, we get $\alpha = \sqrt{-\frac{1}{3}t\exp\left[\frac{1}{A_f}\int\frac{1}{Q(x)}dx\right]}$, from which we get,

$$G(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^{\frac{3}{2}}$$ and

$$G(x,t) = G(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{\frac{3}{2}}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right]$$ as before which

are t and x-evolutions respectively of gluon structure function for $\beta = \alpha^3$.

Proceeding exactly in the same way, we can show that if we take $\beta = \alpha^4$ we get

$$G(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^{\frac{4}{3}}$$ and

$$G(x,t) = G(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{\frac{4}{3}}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right]$$ and so on. So, in

general, if we take $\beta = \alpha^y$, we get

$$G(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^{\frac{y}{y-1}}$$ and

$$G(x,t) = G(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{\frac{y}{y-1}}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right],$$ which give t and

x-evolutions respectively of gluon structure function for $\beta = \alpha^y$. We observe if $y\rightarrow\infty$ (very large), $y/(y-1) \rightarrow 1$.

Thus we observe that if we take $\beta = \alpha$ in equation (11) we can not obtain the value of α and also the required solution. But if we take $\beta = \alpha^2, \alpha^3, \alpha^4, \alpha^5$….. and so on, we see that the powers of (t/t_0) in t-evolutions of gluon structure functions are 2, 3/2, 4/3, 5/4….and so on respectively as discussed above. Similarly, for x-evolutions of gluon structure functions we see that the numerators of the first term inside the integral sign are 2, 3/2, 4/3, 5/4….and so on respectively for the same values of α. Thus we see that if in the relation $\beta = \alpha^y$, y varies between 2 to a maximum value, the powers of (t/t_0) and the numerators of the first term in the integral sign vary between 2 to 1. Then it is understood that the solution of equations (9) obtained by this methodology is not unique and so the t and x-evolution of gluon structure function obtained by this methodology is not unique. Thus by this methodology, instead of having a single solution we arrive a band of solutions, of course the range for these solutions is reasonably narrow.

Again, for Q^2 values much larger than Λ^2, the effective coupling is small and a perturbative description in terms of quarks and gluons interacting weakly makes sense. For Q^2 of order Λ^2, the effective coupling is infinite and we cannot make such a picture, since quark and gluons will arrange themselves into strongly bound clusters, namely, hadrons [16]. Also the perturbation series breaks down and structure functions must vanish [17]. Thus, Λ can be considered as the boundary between a world of quasi-free quarks and gluons, and the world of pions, protons, and so on. The value of Λ is not predicted by the theory; it is a free parameter to be determined from experiment. It should expect that it is of the order of a typical hadronic mass [16]. The value of Λ is so small that we can take at $Q = \Lambda$, $F_2^{S}(x, t) = 0$ due to conservation of the electromagnetic current [18]. Since the relation between gluon and singlet structure function is $G(x, t) = K_1(x)F_2^S(x, t)$, therefore $G(x, t) = 0$ at $Q = \Lambda$. Using this boundary condition in equations (11) we get $\beta = 0$ and

$$G(x,t) = \alpha t\exp\left[\int\left(\frac{1}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right]. \tag{16}$$

Now, defining $G(x,t_0) = \alpha t_0\exp\left[\int\left(\frac{1}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right]$, at $t = t_0$, where $t_0 = \ln(Q_0^2/\Lambda^2)$ at any lower value $Q = Q_0$, we get from equation (16)

$$G(x,t) = G(x_0,t)\left(\frac{t}{t_0}\right), \tag{17}$$

which gives the t-evolution of gluon structure function $G(x, t)$ in LO. Again defining,

ISSN 2229-5518

$$G\left(x_0,t\right)=\alpha t\exp\left[\int\left(\frac{1}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right]_{x=x_0}$$

, we obtain from equation (16)

$$G(x,t)=G\left(x_0,t\right)\exp\left[\int_{x_0}^{x}\left(\frac{1}{A_f Q(x)}-\frac{P(x)}{Q(x)}\right)dx\right], \tag{18}$$

which gives the x-evolution of gluon structure function $G(x, t)$ in LO. We observed that unique solutions (equations (17) and (18)) of DGLAP evolution equation for gluon structure function are same with particular solutions for y maximum in $\beta = \alpha^y$ relation in LO.

3. Results and discussion

In the present paper, we present our result of t-evolution of gluon structure function qualitatively and compare result of x-evolution with a recent global parameterization [7-8]. These parameterizations include data from H1, ZEUS, DO, CDF experiment. Though we compare our results with $y = 2$ and y = maximum in $\beta = \alpha^y$ relation with the parameterization, our result with y = maximum is equivalent to that of unique solution.

In figure 1(a-b), we present our results of t-evolutions of gluon structure functions $G(x, t)$ qualitatively for the representative values of x given in the figures for $y = 2$ (upper solid and dashed lines) and y maximum (lower solid and dashed lines) in $\beta = \alpha^y$ relation. We have taken arbitrary inputs from recent global parameterizations MRST2001 (solid lines) and MRST2001J (dashed lines) in figure 1(a) at $Q_0^2 = 1\ \text{GeV}^2$ [7] and MRS data in figure 1(b) at $Q_0^2 = 4\ \text{GeV}^2$ [8]. It is clear from figures that t-evolutions of gluon structure functions depend upon input $G(x, t_0)$ values.

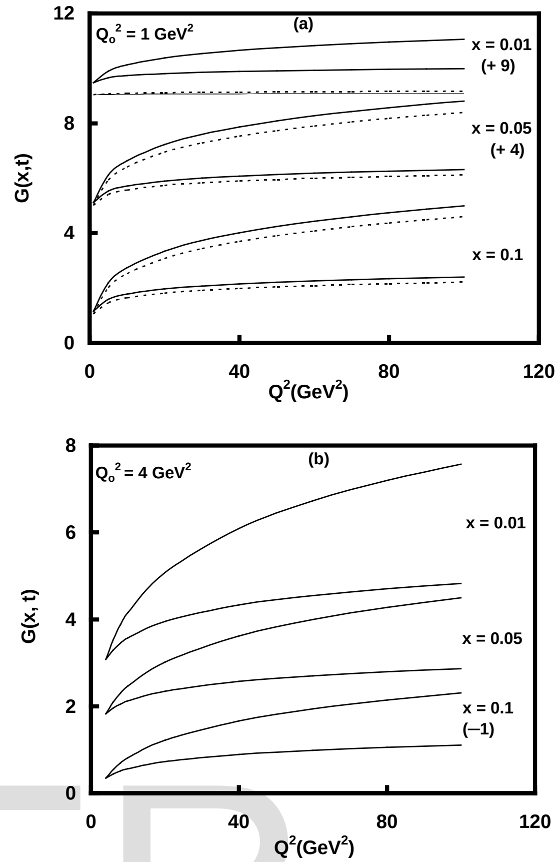

Fig. 1(a-b): Results of t-evolutions of gluon structure functions for the representative values of x given in the figures for $y = 2$ (upper solid and dashed lines) and y maximum (lower solid and dashed lines) in $\beta = \alpha^y$ relation. We have taken arbitrary inputs from recent global parameterizations MRST2001 (solid lines) and MRST2001J (dashed lines) in figure 1(a) and MRS data in figure 1(b) at $Q_0^2 = 1\ \text{GeV}^2$ and $Q_0^2 = 4\ \text{GeV}^2$ respectively. For convenience, value of each data point is increased by adding 9 and 4 for $x = 0.01$ and $x = 0.05$ respectively in figure 1(a) and decreased by subtracting 1 for $x = 0.1$ in figure 1(b).

For a quantitative analysis of x-distributions of gluon structure functions $G(x, t)$, we calculate the integrals that occurred in equation (15) for $N_f = 4$. In figure 2(a-b), we present our results of x-distribution of gluon structure functions for $K_1(x) = ax^b$, where 'a' and 'b' are constants, for representative values of Q^2 given in each figure, and compare them with recent global parameterizations [7] for y minimum in the relation $\beta=\alpha^y$. In figure 2(a), we observe that agreement of the results with parameterization is found to be very poor for any values of 'a' and 'b' at low-x and agreement is found to be good at high-x at $a = 372$ and $b = 4$ (thick solid line). In figure 2(b), agreement of the results with parameterizations is found to be good at $a = 135$ and b

ISSN 2229-5518

= 1.8 (thick solid line) in $\beta = \alpha^y$ relation. In the same figures we present the sensitivity of our results for different values of 'a' at fixed value 'b'. Here we take $b = 4$ in figure 2(a) and

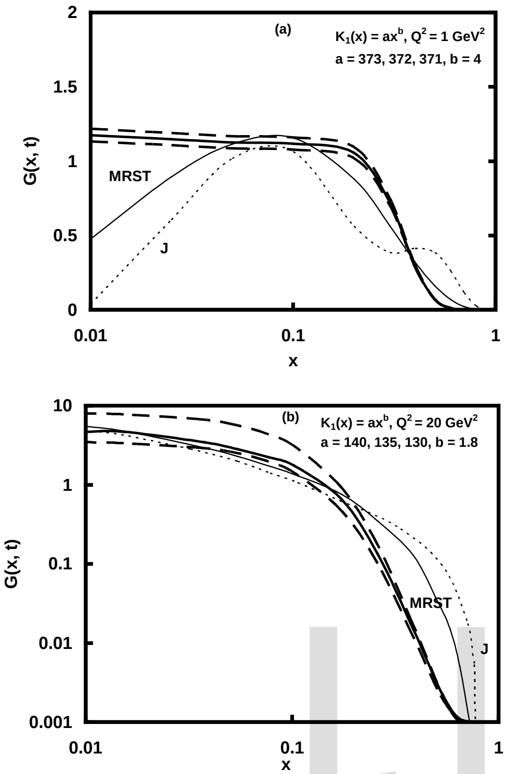

Fig. 2(a-b): Results of x-distribution of gluon structure functions for $K_1(x) = ax^b$, where 'a' and 'b' are constants for representative values of Q^2 given in each figure, and compare them with recent global parameterizations for y minimum (thick solid lines) in the relation $\beta = \alpha^y$. In the same figures we present the sensitivity of our results for different values of 'a' at fixed value 'b'. Here we take $b = 4$ in figure 2(a) and $b = 1.8$ in figure 2(b).

$b = 1.8$ in figure 2(b). We observe that if value of 'a' is increased or decreased, the curve goes upward or downward direction respectively. But the nature of the curves is similar. Here thin solid and dotted lines are MRST 2001 and MRST2001J [7] parameterizations.

In figure 3(a-b), we present the sensitivity of our results for different values of 'b' at fixed value of 'a'. Here we take $a = 372$ in figure 3(a) and $a = 135$ in figure 3(b). We observe that, agreement of the results (thick solid lines) with parameterizations is good in figure 3(a) at b = 4 and figure 3(b) at $b = 1.8$. If value of 'b' is increased or decreased the curve goes downward or upward directions. But the nature of the curve is similar.

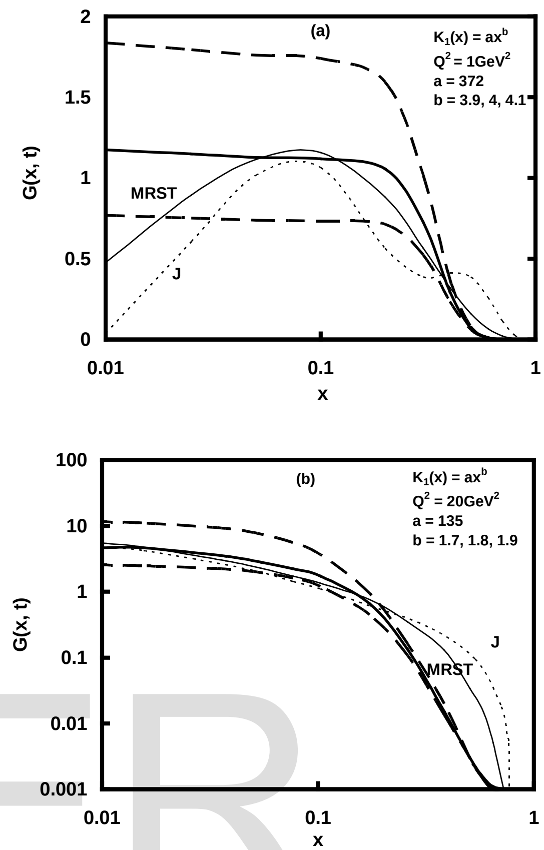

Fig. 3(a-b): Sensitivity of our results for different values of 'b' at fixed value of 'a'. Here we take $a = 372$ in figure 3(a) and $a = 135$ in figure 3(b).

In figure 4(a-b), we present our results of x-evolution of gluon structure function $G(x, t)$ for $K_1(x) = ax^b$ for y minimum (lower thick solid lines) and maximum (upper thick solid lines) in relation $\beta = \alpha^y$ at same parameter values $a = 372$, $b = 4$ in figure 4(a) and $a = 135$, $b = 1.8$ in figure 4(b) and for representative values of Q^2 given in each figure, and compare them with recent global parameterizations [7]. We observe that result of x-evolution of gluon structure function for y maximum (long dashed lines) coincide with result of x-evolution of gluon structure function for y minimum (lower thick solid lines) when $a = 375$, $b = 4.7$ in figure 4(a) and $a = 134$, $b = 2$ in figure 4(b). That means if y varies from minimum to maximum, then value of parameter 'a' varies from 372 to 375 and 'b' varies from 4 to 4.7 in figure 4(a) and 'a' varies from 135 to 134 and 'b' varies from 1.8 to 2 in figure 4(b).

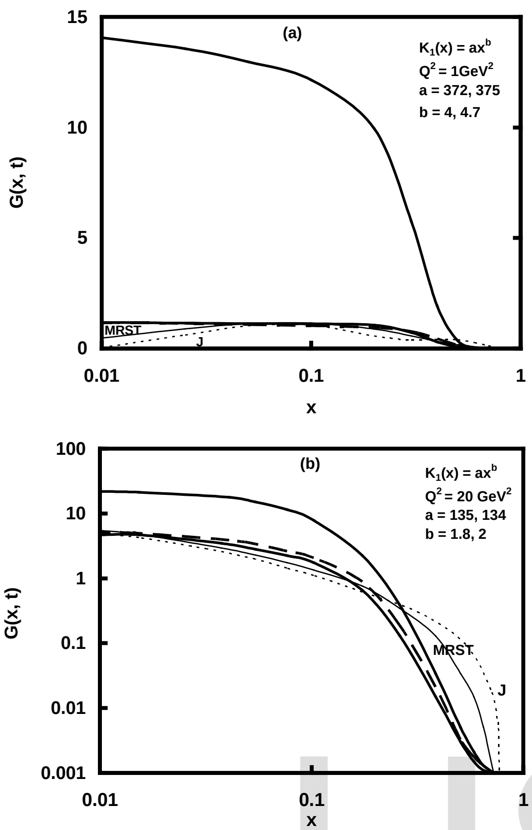

Fig. 4(a-b): Results of x-evolution of gluon structure function $G(x, t)$ for $K_1(x) = ax^b$ for y minimum (lower thick solid lines) and maximum (upper thick solid lines) in relation $\beta = \alpha^y$ at same parameter values $a = 372$, $b = 4$ in figure 4(a) and $a = 135$, $b = 1.8$ in figure 4(b) and for representative values of Q^2 given in each figure, and compare them with recent global parameterizations. Result of x-evolution of gluon structure function for y maximum (long dashed lines) coincide with result of x-evolution of gluon structure function for y minimum (lower thick solid lines) when $a = 375$, $b = 4.7$ in figure 4(a) and $a = 134$, $b = 2$ in figure 4(b).

In figure 5(a-b), we present our results of x-distribution of gluon structure functions $G(x, t)$ for $K_1(x) = ce^{-dx}$, where 'c' and 'd' are constants for representative values of Q^2 given in each figure, and compare them with recent global parameterizations [7] for y minimum in the relation $\beta = \alpha^y$. In figure 5(a), we observe that agreement of the results with parameterization is found to be very poor for any values of 'c' and 'd' at low-x and agreement is found to be good at high-x at $c = 300$ and $d = -3.8$ (thick solid line). In figure 5(b) agreement of the results with parameterizations is found to be good at $c = 5$ and $d = -28$ (thick solid line). In the same figures, we present the sensitivity of our results for different values of 'c' by thick dashed lines at fixed value'd'.

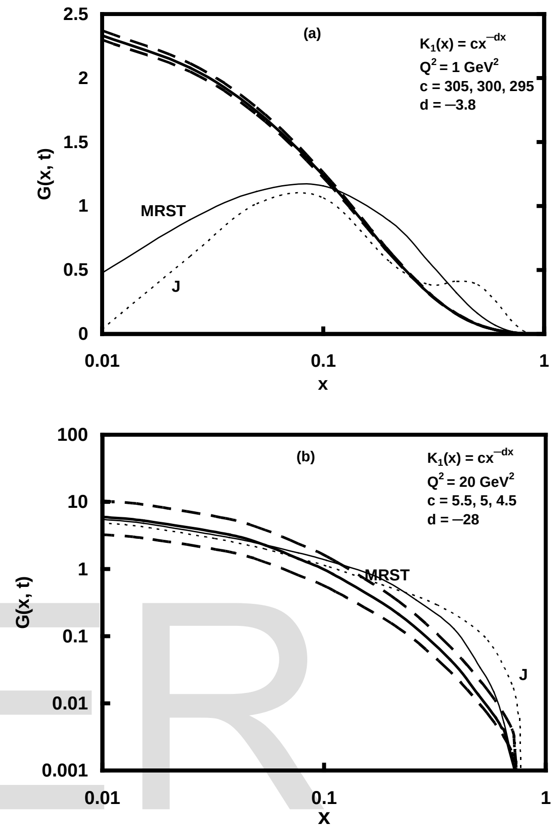

Fig. 5(a-b): Results of x-distribution of gluon structure functions $G(x, t)$ for $K_1(x) = ce^{-dx}$, where 'c' and 'd' are constants for representative values of Q^2 given in each figure, and compare them with recent global parameterizations for y minimum in the relation $\beta = \alpha^y$. In the same figures we present the sensitivity of our results for different values of 'c' by thick dashed lines at fixed value 'd'. Here we take $d = -3.8$ in figure 5(a) and $d = -28$ in figure 5(b).

Here we take $d = -3.8$ in figure 5(a) and $d = -28$ in figure 5(b). We observe that if value of 'c' is increased or decreased, the curve goes upward or downward directions respectively. But the nature of the curve is similar.

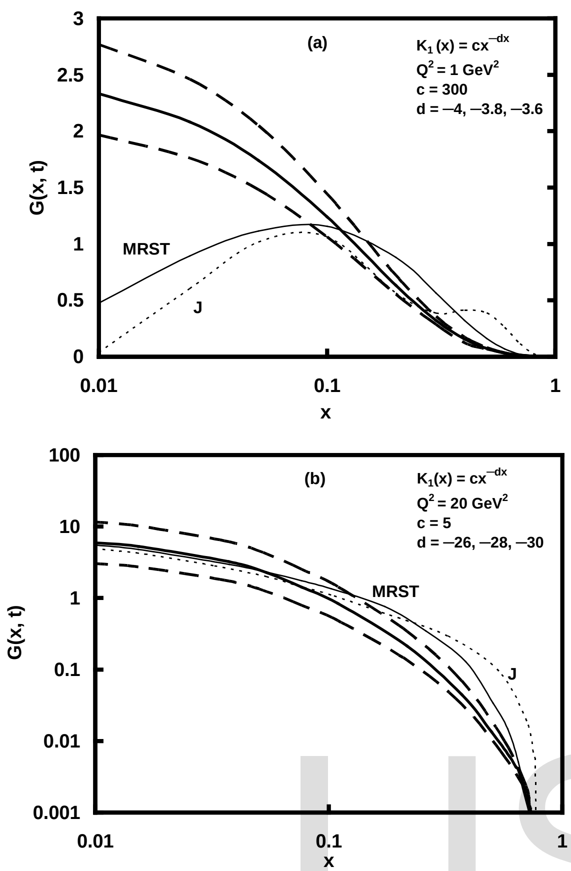

Fig. 6(a-b): Sensitivity of our results for different values of 'd' at fixed value of 'c'. Here we take c = 300 in figure 6(a) and c = 5 in figure 6(b).

In figure 6(a-b), we present the sensitivity of our results for different values of 'd' at fixed value of 'c'. Here we take c = 300 in figure 6(a) and c = 5 in figure 6(b). We observe that agreement of the results (thick solid lines) with parameterizations is good in figure 6(a) at d = -3.8, and 6(b) at d = -28. If value of 'd' is increased or decreased, the curve goes downward or upward direction in figure 6(a) and if value of 'd' is increased or decreased the curve goes upward or downward direction in figure 6(b) . But the nature of the curves is similar in both cases.

In figure 7(a-b), we present our results of x-evolution of gluon structure function $G(x, t)$ for $K_1(x) = ce^{-dx}$ for y minimum (lower thick solid lines) and maximum (upper thick solid lines) in the relation $\beta = \alpha^y$ at same parameter values c = 300, d = -3.8 i nfigure 7(a) and c = 5, d = -28 in figure 7(b) and for representative values of Q^2 given in each figure, and compare them with recent global parameterizations [7]. We observe that result of x-evolution

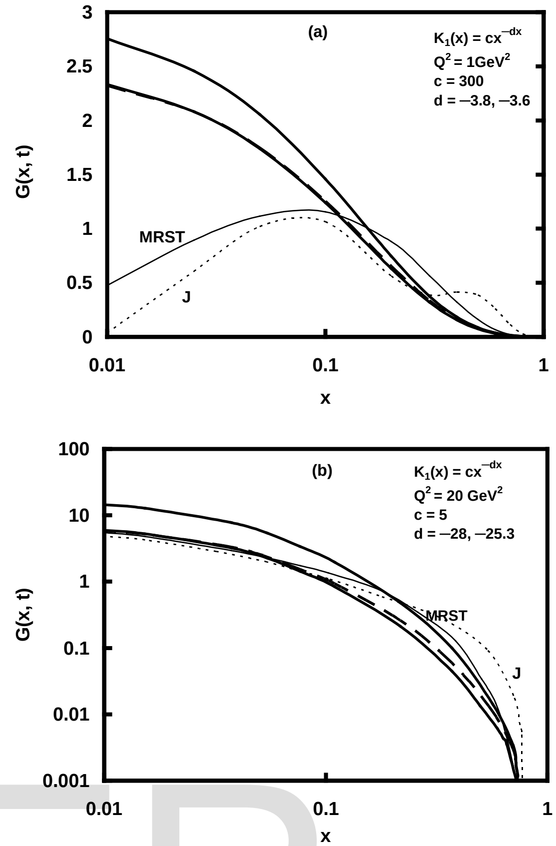

Fig. 7(a-b): Results of x-evolution of gluon structure function $G(x, t)$ for $K_1(x) = ce^{-dx}$ for y minimum (lower thick solid lines) and maximum (upper thick solid lines) in the relation $\beta = \alpha^y$ at same parameter values c = 300, d = -3.8 in figure 7(a) and c = 5, d = -28 in figure 7(b) and for representative values of Q^2 given in each figure, and compare them with recent global parameterizations. Result of x-evolution of gluon structure function, for y maximum (long dashed lines) coincide with result of x-evolution of gluon structure function for y minimum (lower thick solid lines) when c = 300, d = -3.6 in figure 7(a) and c = 5, d = -25.3 in figure 7(b).

of gluon structure function, for y maximum (long dashed lines) coincide with result of x-evolution of gluon structure function for y minimum (lower thick solid lines) when c = 300, d = -3.6 in figure 7(a) and c = 5, d = -25.3 in figure 7(b). That means if y varies from minimum to maximum, then value of parameter 'd' varies from -3.8 to -3.6 in figure 7(a) and from -28 to -25.3 in figure 7(b). In these cases, value of parameter 'c' remains constant. It is to be noted that agreement of the results with parameterization is found to be very poor for any constant value of $K_1(x)$. Therefore, we do not present our result of x-distribution at $K_1(x)$ = constant. Moreover, in general, the agreement of our results with the parameterization at small-x is poor for low-Q^2

ISSN 2229-5518

value and excellent for high-Q^2 value which is quite expected.

From our above discussion, it has been observed that though we can derive a unique t-evolution for gluon structure function in LO, yet we can not establish a unique x-evolution for gluon structure function in LO. $K_1(x)$, the relation between singlet and gluon structure functions, may be in the forms of a constant, an exponential function of x or a power in x and they can equally produce required x-distribution of gluon structure functions. But unlike many parameter arbitrary input x-distribution functions generally used in the literature, our method requires only one or two such parameters. On the other hand, The explicit form of $K_1(x)$ can actually be obtained only by solving coupled DGLAP evolution equations for singlet and gluon structure functions, and works are going on in this regard.

4. Conclusion

In this paper, we obtain complete and unique solutions of gluon distribution function at low-x using Taylor's expansion method from GLDAP evolution equations and t and x-evolution of gluon structure functions in leading order. We compare our results with a global parameterization. In all the results from global fits, it is seen that, gluon structure functions increases when x decreases and Q^2 increases for fixed values of Q^2 and x respectively. It has been observed that, though we have derived a unique t-evolution for gluon in LO, yet we can not establish a completely unique x-evolution for gluon structure functions in LO due to the relation $K_1(x)$ between singlet and gluon structure functions. $K_1(x)$ may be in the forms of an exponential function or a power function and they can equally produce required x-distribution of gluon structure functions. But unlike many parameter arbitrary input x-distribution functions generally used in the literature, our method requires only one or two such parameters.

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